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PRICE CAPS WITH CAPACITY PRECOMMITMENT*

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Abstract

Contrary to a recent literature that questions whether price caps are effective, and even sensitive, under demand uncertainty, we show that in the absence of quantity precommitment the effects of a price cap are the same whether the demand is uncertain or deterministic. Next we study the effectiveness of a price cap to regulate a monopoly that makes irreversible capacity investments ex-ante, and then chooses its output up to capacity upon observing the realization of demand. In this more interesting scenario the optimal price cap, which must trade off the incentives for capacity investment and capacity withholding, is well above the unit cost of capacity, and may be below the price cap that maximizes capacity. Further, under standard assumptions on the demand distribution the comparative static properties of price caps above the optimal price cap are analogous to those they have in the absence of capacity precommitment. Nevertheless, a price cap alone cannot eliminate inefficiencies.

Keywords: Monopoly, Market Power, Price Cap Regulation, Capacity Investment, Capacity Withholding, Demand Uncertainty.

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1 Introduction

Since Littlechild (1983)'s report, price cap regulation is regarded as an effective instrument to mitigate market power, foster cost minimization, and ultimately enhance surplus: When precise information about cost and demand is available, the introduction of a binding price cap rises firms' marginal revenue near the equilibrium output and leads to an increase of the equilibrium output and surplus, and to a decrease of the market price. Moreover, under broad regularity conditions on the demand and cost functions, for any price cap above marginal cost both output and surplus decrease, and the market price increases with the price cap. Further, in the most favorable conditions (e.g., when firms produce the good with constant returns to scale), a price cap equal to marginal cost is able to eliminate inefficiencies. (In contrast, rate-of-return regulation, used for most of the 20th century to regulate public utilities, distorts incentives for cost minimization – see, e.g., Joskow (1972) – or cost reduction – see, e.g., Cabral and Riordan (1989).)

We study the effectiveness of price cap regulation under demand uncertainty.[1] We begin by showing that in the absence of quantity precommitment, e.g., when the good can be produced instantly upon the realization of demand or there is slack capacity, the effects of price caps remain exactly the same as when the demand is deterministic. These preliminary results naturally raise the question of how price caps affect capacity decisions.

In order to tackle this issue, we consider a setting in which a monopolist makes irreversible capacity investments ex-ante, and then chooses its output up to capacity upon observing the realization of demand. Thus, the monopolist may withhold capacity if it finds it beneficial to do so. Capacity withholding is common in many important markets such as sport events, hotel accommodation, electricity, or agricultural products.[2] In this setting, inefficiencies arise both because the monopolist

[1] Demand uncertainty may be interpreted also as variations of demand over time, as is common in electricity markets – see, e.g., Green and Newbery (1992).

[2] In electricity markets firms may declare some of their generators to be unavailable – data for the California electricity market during the time period May 2000-December 2001 show that at the price cap some generators did not supply all of their uncommitted capacity – see Cramton (2003) and Joskow and Kahn (2002). In markets for agricultural products, farmer associations sometimes destroy part of the output.

installs a low level of capacity in order to precommit to high prices, and because the monopolist withholds capacity for low demand realizations in order to avoid prices to fall too low.[3]

The effects of price cap regulation with demand uncertainty and capacity precommitment and withholding are subtle. We show that, much as in the absence of capacity precommitment, the introduction of a (sufficiently large) binding price cap raises the firms' marginal return to capacity investment near the equilibrium capacity and leads to an increase of the equilibrium capacity, the expected output and the expected total surplus, and to a decrease of the expected market price. However, price caps near the unit cost of capacity are suboptimal because they reduce the return to capacity investment below its cost, and lead the monopolist to install no capacity. The optimal price cap (i.e., the price cap that maximizes surplus) must trade off appropriately the incentives for capacity investment and capacity withholding, and tends to be well above the unit cost of capacity. When the unit cost of capacity is high the effect on capacity investment is dominant, and the optimal price cap maximizes capacity investment. When the unit cost of capacity is low, reducing the price cap below the level that maximizes capacity investment increases expected surplus. (Thus, in this case maximizing capacity investment does not warrant maximizing expected surplus.)

With capacity precommitment and withholding the comparative static properties of price caps are more complex than in the (static) setting in which the monopolist can produce an arbitrary output upon the realization demand. Under standard regularity assumptions on the demand distribution, the effects of changes in the price cap on expected output and surplus depend on the magnitude of its effects on capacity investment and capacity withholding, which have opposite signs. Capacity investment is a single peaked function of the price cap: for low price caps it increases with the price cap until it reaches a maximum at some binding price cap r^*, and then decreases with the price cap above r^*. When the unit cost of capacity is large the signs of the

[3] Focusing on the monopolistic case allows us to avoid some potential conundrums that arise in dynamic oligopolistic settings, which are distractions from the issue under scrutiny – the impact of price cap regulation. For example, it is unclear what is the appropriate model of competition to consider at the ex-post stage. Moreover, when demand is uncertain there are well known difficulties therein to guarantee existence, uniqueness and symmetry of equilibrium – see, e.g., Reynolds and Wilson (2000), Gabszewicz and Poddar (1997).

effects of changes in the price cap on expected output, expected surplus and capacity investment coincide. Interestingly, when the unit cost of capacity is small, expected output and expected surplus decrease with the price cap above and around r^*, and thus the optimal price cap is below r^*. Price caps affect the market price directly, but also indirectly via their impact on the level of capacity. Thus, an increase of the price cap increases the expected price above and around r^*, but has an ambiguous effect below r^*.

Thus, a binding price cap actually provides incentives for capacity investment. Further, it discourages capacity withholding. Nonetheless, a price cap alone is unable to provide the appropriate incentives for capacity investment and simultaneously eliminate the inefficiencies arising from capacity withholding: the optimal price cap induces a low level of capacity, and does not prevent capacity withholding. Hence, with demand uncertainty and capacity precommitment an optimal regulatory policy may require using other instruments – e.g., forward markets or subsidies to capacity.

Earle et al. (2007) studies an oligopolistic model in which firms make output decisions ex-ante, i.e., firms choose their output before the realization of demand and supply it inelastically and unconditionally. In this setting, it shows that for price caps near marginal cost the output is suboptimally low and may increase with the price cap. Moreover, it establishes that the comparative static properties of price caps that hold when the demand is deterministic fail for a generic stochastic demand schedule.[4] These results lead Earle et al. (2007) to conclude that the "standard arguments supporting the imposition of price caps break down in the presence of demand uncertainty."

This sweeping conclusion of Earle et al. (2007) is unfounded. Moreover, the source of its results is not demand uncertainty *per se*, but it is demand uncertainty in conjunction with quantity precommitment (which is implicitly assumed in its model): As we show, in the absence of quantity (or capacity) precommitment the properties of price caps are the same whether the demand is deterministic or stochastic. Further, in

[4] Specifically, Earle at al. (2007) shows that for any demand distribution such that output decreases with the price cap at a given binding price cap r, it is possible to perturb the demand distribution on an arbitrarily small interval (shifting the probability on the interval to the end-points, thus creating two atoms) in such a way that with this perturbed demand distribution output increases with the price cap near r.

the more interesting setting studied in the present paper, in which capacity decisions are made ex-ante and output decisions are made ex-post, we show that a price cap is an effective regulatory instrument to provide incentives for capacity investment and to discourage capacity withholding, and that under standard regularity assumptions on the demand distribution the comparative static properties of price caps are analogous to those arising when capacity has no precommitment value. (In addition, the proof of Earle et al. (2007)'s Theorem 6, claiming that its genericity result applies to the present setting, is incorrect. See Lemus and Moreno (2014).[5])

Other authors have studied price cap regulation in the presence of exogenous technological progress – in our setting the unit cost of capacity and production are constant over the regulatory period. Biglaiser and Riordan (2000), for example, study the incentive properties of price cap to produce optimal capacity investment and replacement. In their setting, they find that price caps provide better incentives than rate-of-return regulation, although in their setting (as in ours) optimal price caps must deal with a trade off involving the incentives for capacity investment and replacement. In an oligopolistic industry, Roques and Savva (2009) study the effect of price caps on the timing of investments when demand is uncertain, and find that as in our setting a low price cap may be suboptimal as it may disincentivize investment. Reynolds and Rietzke (2012) study the impact of price caps in oligopolistic markets with endogenous entry, and identify conditions under which a price cap improves welfare. Dobbs (2004) studies the effect intertemporal price cap regulation when a monopolist facing demand uncertainty has to decide the size and timing of its investments, and shows that optimal price caps lead to under investment and quantity rationing. Dixit (1991) studies a competitive market in which demand is uncertain and firms make ex-ante irreversible investments, and shows that introducing price ceilings lead to delay investments and higher prices over time.

The paper is organized as follows: In Section 2 we study the effects of price caps in the absence of quantity precommitment. In Section 3 we describe the more

[5] Moreover, Grimm and Zoettl (2010) shows that under standard regularity assumptions the comparative static properties of price caps are recovered also in the setting where firms cannot withhold capacity. This paper also provides a reduced form analysis of an oligopolistic setting where firms may withhold capacity, but mistakenly concludes that maximizing the expected surplus amounts to maximizing capacity. In particular, its equation (5) providing the marginal revenue is incorrect in region A – see Section 3.

interesting setting in which a monopoly makes irreversible capacity investments ex-ante and then chooses its output up to capacity upon the realization of demand, and we derive the equilibrium capacity as a function of the price cap. In Section 4 we study the comparative static properties of price caps in this setting. In Section 5 we study optimal price caps. We discuss an example in Section 6, and we conclude in Section 7. Appendix A contains the proofs. Appendix B studies the model of full capacity utilization, and discusses the differing results obtained in that setting.

2 Price Caps without Quantity Precommitment

Consider a monopoly that produces a good with constant returns to scale and unit cost $c \in \mathbb{R}_+$. The market demand is known with certainty and is given by $D(x, p) = \max\{x - p, 0\}$, where $p \in \mathbb{R}_+$ is the market price, and $x \in \mathbb{R}_+$ is the consumers' maximum willingness to pay for the good. In the monopoly equilibrium the output q^* and price p^* are given by $q^*(x) = 0$ and $p^*(x) \geq x$ if $x < c$, and by

$$q^*(x) = \frac{x - c}{2} \text{ and } p^*(x) = \frac{x + c}{2},$$

otherwise. (Writing the price and output as functions of x will be useful in the sequel.) The effect of a price cap $r \in \mathbb{R}_+$ on the monopoly equilibrium is well known. Denote by P and Q the functions providing the price and output in the monopoly equilibrium for each (r, x). A low price cap $r < c$ leads the monopoly to serve no output, i.e., $Q(r, x) = 0$, whereas a high (non-binding) price cap $r \geq p^*(x)$ has no effect on the monopoly equilibrium, i.e., $Q(r, x) = q^*(x)$ and $P(r, x) = p^*(x)$. An intermediate price cap $r \in [c, p^*)$, however, increases the monopolist's marginal revenue around $q^*(x)$, and leads to an increase of the monopolist's output to $Q(r, x) = x - r > q^*(x)$, and to a decrease of the market price to $P(r, x) = r < p^*(x)$, see Figure 1. Thus, a decrease of the price cap r on (c, p^*) leads to an increase of the output and the surplus, and to a decrease of the market price. Hence, a price cap $r = c$ maximizes the output as well as the surplus.

Let us consider now the case of demand uncertainty. (As noted above, demand uncertainty may be interpreted also as variations of demand over time – see, e.g., Green and Newbery (1992).) Assume that the demand is the function $D(x, p)$ given above but x is now the realization of a random variable X with *p.d.f.* f and support

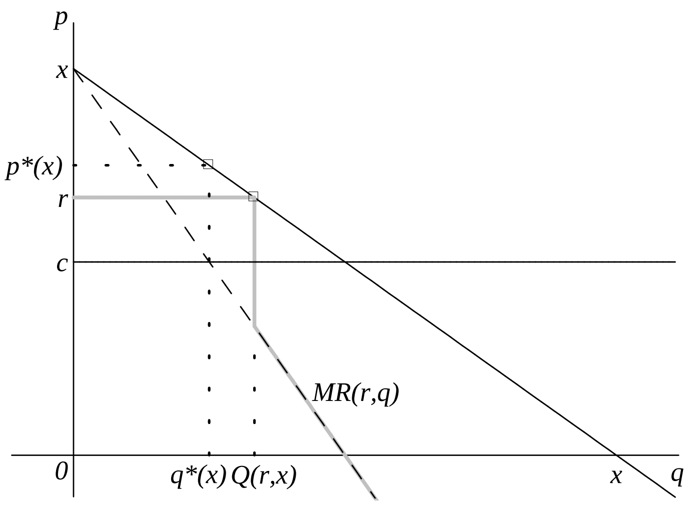

Figure 1: The Effect of a Price Cap with a Deterministic Demand

on a bounded interval $[\alpha, \beta] \subset \mathbb{R}_+$, where $\beta > c$. Studying the impact of a price cap under demand uncertainty requires to specify the timing of decisions. Let us consider a simple model in which the monopolist decides its output upon observing the realization of demand.

In the absence of a price cap, for each demand realization $x \in [\alpha, \beta]$ the monopoly equilibrium output and price are given by $q^*(x)$ and $p^*(x)$ defined above. Write

$$P^* = \max_{x \in [c,\beta]} p^*(x) = (\beta + c)/2.$$

The introduction of a price cap $r \in \mathbb{R}_+$ has a simple effect on the monopoly equilibrium: a low price cap $r < c$ leads the monopoly to serve no output regardless of the realization of demand, i.e., $Q(r, \cdot) = 0$. A high (non-binding) price cap $r \geq P^*$, results in an output $Q(r, \cdot) = q^*(\cdot)$ and price $P(r, \cdot) = p^*(\cdot)$. An intermediate price cap $r \in [c, P^*)$ is non-binding for low demand realizations, but becomes binding for high demand realizations – see Figure 2.

The effects of a price cap r on the monopolist's output and price for each demand realization are described in Table 1 below.

X	$[\alpha, 2r - c)$	$[2r - c, \beta]$
$P(r, x)$	$p^*(x)$	r
$Q(r, x)$	$q^*(x)$	$x - r$

Table 1: Equilibrium Output and Price for $r \in [c, P^*)$.

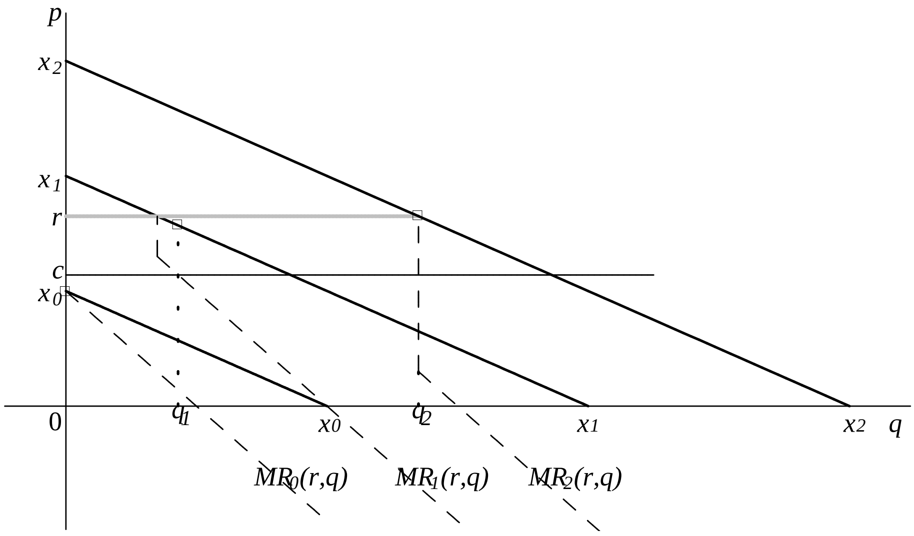

Figure 2: The Effect of a Price Cap with Demand Uncertainty

Hence the expected output $E(Q(r,X))$ is given for $r \in (c, P^*)$ by

$$E(Q(r,X)) = \frac{1}{2}\int_c^{2r-c} (x-c)\, f(x)dx + \int_{2r-c}^{\beta} (x-r)\, f(x)dx.$$

Differentiating this expression, and noting that $\beta > 2r - c$ for $r < P^*$, yields

$$\frac{dE(Q(r,X))}{dr} = -\int_{2r-c}^{\beta} f(x)dx < 0.$$

Thus, as in the case of demand certainty the expected output and the expected surplus decrease with the price cap on (c, P^*). The expected price is not well defined since for $x < c$ the monopolist supplies no output. However, decreasing the price cap decreases the market price for demand realizations $x > 2r - c$, and has no effect on the market price for demand realizations $x \in (c, 2r - c)$, and therefore unambiguously decreases the expected price over the realizations in which there is trade. Thus, when demand is stochastic setting a price cap r equal to the unit cost of production c maximizes the expected output as well as the expected surplus, just as in the case of a deterministic demand. We summarize these findings in Proposition 1. These results can be easily extended to a Cournot oligopolistic setting.

Proposition 1. *In the absence of quantity precommitment a binding price cap r above the marginal cost c leads to an increase of the equilibrium expected output and expected surplus, and to a decrease of the expected price. Moreover, the expected*

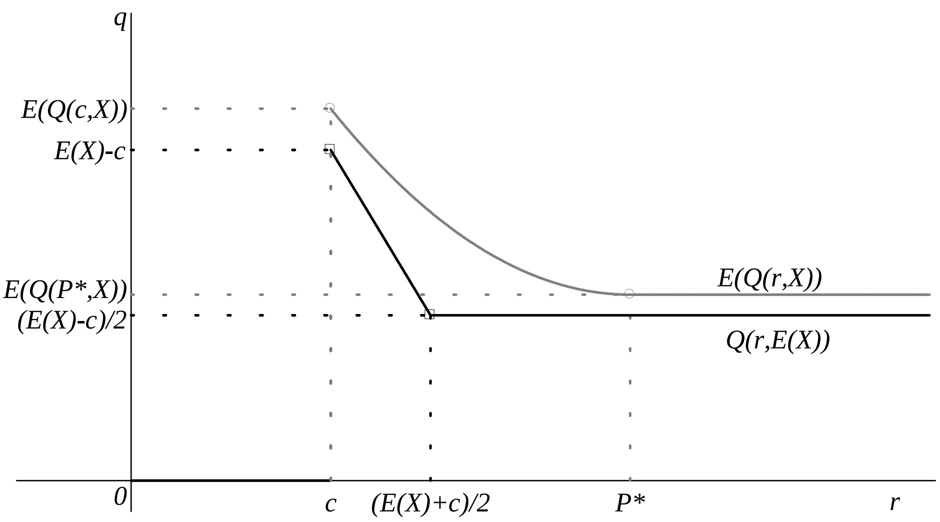

Figure 3: The Effect of Price Caps on Output without Quantity Precommitment.

output and the expected surplus decrease with r, *whereas the expected price increases with* r. *Further,* $r^* = c$ *maximizes the expected surplus.*

Figure 3 illustrates the conclusions of Proposition 1. The expected output of the monopoly as a function of the price cap $E(Q(r, X))$ is calculated assuming that X is distributed uniformly on $[0, 1]$. The figure also shows the output of a monopoly facing a known demand $D(E(X), p)$. As Figure 3 shows price caps have qualitatively the same effects whether the demand is deterministic or stochastic – the only effect of uncertainty is smoothing the non-differentiability at the lowest non-binding price cap arising when the demand is deterministic. Thus, price cap regulation is an effective instrument to mitigate market power and foster efficiency whether the demand is deterministic or stochastic.

This analysis is useful when firms are not capacity constrained (or when capacity can be installed instantly). Relevant examples are the Spanish or California electricity markets, in which (at least in recent times) firms have excess capacity and their bids are short lived (i.e., firms compete to serve the demand for short periods of time, e.g., for hourly or half hourly periods). Of course, price cap regulation has an impact on firms' capacity investments, which are long run decisions made prior the realization of demand. Thus, endogenizing firms' capacity investment decisions seems a natural next step to take.

3 Capacity Precommitment and Withholding

In what follows we study the impact of price caps in a model in which the monopolist makes ex-ante capacity investment decisions and then, upon observing the realization of demand, decides how much to produce, and may withhold capacity if doing so is beneficial. In this setting the level of capacity is a long run decision, whereas the level of output is a short run decision. One may also interpret this setting as if the monopolist decides its output before demand is realized, but once demand is realized the monopolist decides how much to supply, and may supply less than its total output. Relevant examples include the electricity markets mentioned above as well as markets for agricultural products, sport events, hotel accommodation, etc.

Consider a monopolist facing an uncertain demand. As in Section 2, the market demand is given for $p \in \mathbb{R}_+$ by $D(X, p) = \max\{X - p, 0\}$, where X is now a continuous random variable. The monopolist must decide how much capacity to install before the demand is realized. Once capacity is installed the good can be produced with constant returns to scale up to capacity. The monopolist decides its output upon observing the realization of the demand parameter X.

We denote by f and F the *p.d.f.* and *c.d.f.* of X, respectively, and reduce notation by assuming that the support of X is the interval $[0, 1]$.[6] We assume that the cost of installing a unit of capacity is a positive constant c such that $E(X) > c$. (This inequality rules out the trivial cases in which the monopolist installs no capacity.) Also, we assume without loss of generality that the production cost is zero. Propositions 2-7 below identify the effects of price caps in this setting.

Suppose that a regulatory agency imposes a price cap $r \in [0, 1]$. Since the cost of capacity is sunk and the cost of production (up to capacity) is zero, then at the stage of output choice the monopolist maximizes revenue. If the monopolist had an unlimited capacity, then the equilibrium output is as calculated in Section 2 for $c = 0$; i.e., for $x \in [0, 1]$, $Q(r, x) = x - r \leq 1 - r$ if $r < x/2$, and $Q(r, x) = x/2 \leq 1/2$ if $r \geq x/2$. Hence levels of capacity $k > \max\{1 - r, 1/2\}$ are suboptimal since the monopolist would always have idling capacity, and therefore since $c > 0$ may increase its profit

[6] This assumption reduces notation, and facilitates the presentation and the interpretation of our results since the market price is always defined. However, it entails a small loss of generality because the cost of production given capacity and the lower bound of the support of X coincide.

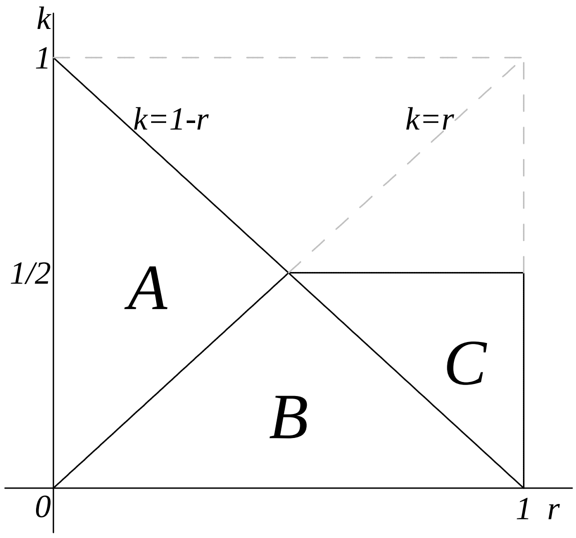

Figure 4: Relevant Price Cap-Capacity Pairs.

by installing less capacity. Thus, we restrict attention to price cap-capacity pairs $(r, k) \in [0, 1]^2$ such that $k \leq \max\{1 - r, 1/2\}$. Figure 4 describes a partition of this set of price cap-capacity pairs into three regions, $A = \{(r, k) \in [0, 1]^2 \mid r \leq k \leq 1-r\}$, $B = \{(r, k) \in [0, 1]^2 \mid k < \min\{1-r, r\}\}$, and $C = \{(r, k) \in [0, 1]^2 \mid 1-r \leq k \leq 1/2\}$.

We calculate the equilibrium price and output in these regions for each realization of the demand parameter X. Abusing notation, we continue to denote by P and Q the price and output, which now are functions of the level of capacity as well as the price cap and the realization of demand. Table 2A describes the prices and output for $(r, k) \in A$.

X	$[0, 2r)$	$[2r, r + k)$	$[r + k, 1]$
$P(r, k, x)$	$x/2$	r	r
$Q(r, k, x)$	$x/2$	$x - r$	k

Table 2A: Equilibrium Output and Price for $(r, k) \in A$.

Figure 5 illustrates the results in Table 2A. For low demand realizations, $x < 2r$, marginal revenue remains positive for levels of output greater than the demand at the price cap, $q = x - r$; hence neither the price cap nor the level of capacity are binding, and therefore the outcome is the unconstrained monopoly equilibrium, i.e., $q = p = x/2$. For intermediate demand realizations, $2r \leq x < r + k$, marginal

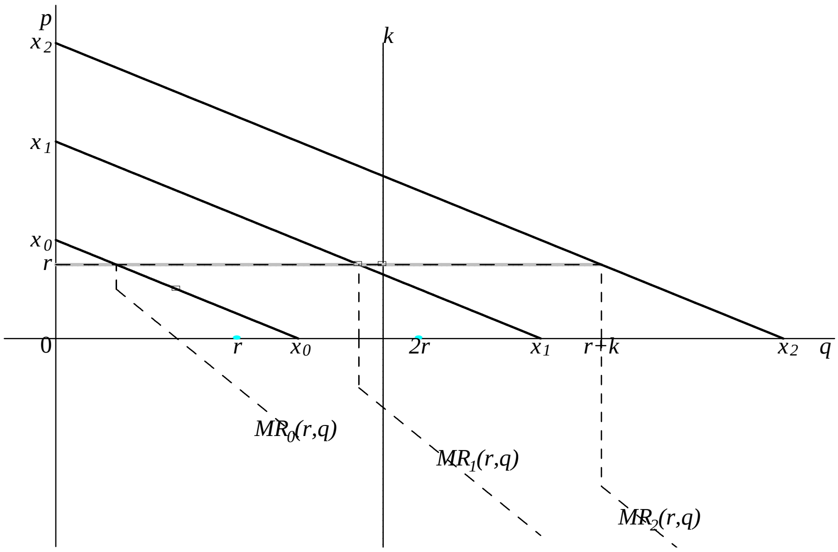

Figure 5: The Effect of a Price Cap when $(r, k) \in A$.

revenue for levels of output greater than $q = x - r$ is negative, and therefore the price cap is binding; hence the monopolist serves the demand at the price cap, and withholds capacity. (Thus, for intermediate demand realizations a marginal decrease of the price cap leads to an increase of output, much as in the models of Section 2.) For high demand realizations, $x \geq r + k$, marginal revenue equals the price cap up to the level of capacity, and hence the monopolist supplies its entire capacity, the price cap remains binding, and the demand is rationed. Note that for price cap-capacity pairs in region A the market price $P(r, k, x)$ is independent of the level of installed capacity k.

Table 2B describes the prices and output for $(r, k) \in B$.

X	$[0, 2k)$	$[2k, r + k)$	$[r + k, 1]$
$P(r, k, x)$	$x/2$	$x - k$	r
$Q(r, k, x)$	$x/2$	k	k

Table 2B: Equilibrium Output and Price for $(r, k) \in B$.

Figure 6 illustrates the results in Table 2B. For low demand realizations $x < 2k$ marginal revenue remains positive for levels of output greater than the demand at the price cap, and therefore neither the price cap nor the level of capacity are binding; hence the outcome is the unconstrained monopoly equilibrium, i.e., $q = p = x/2$. For

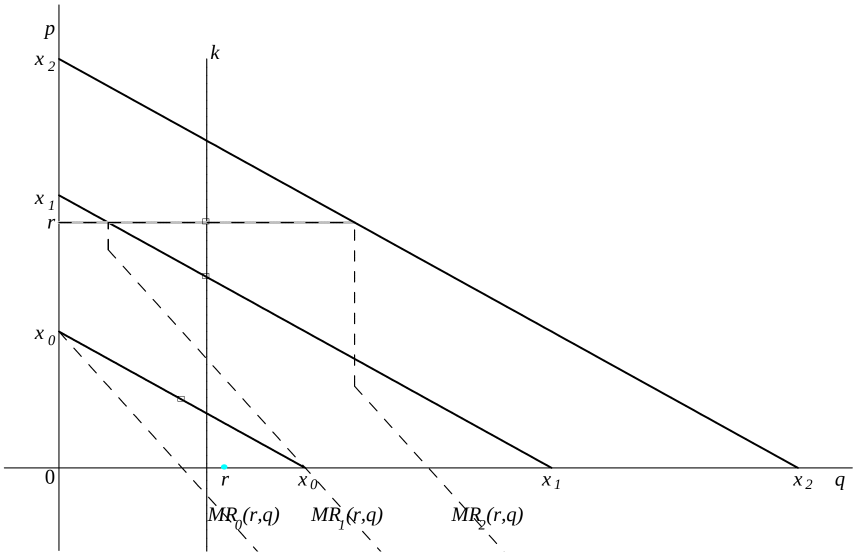

Figure 6: The Effect of a Price Cap when $(r, k) \in B$.

intermediate demand realizations $2k \leq x < r + k$, marginal revenue is positive for output levels greater than k, and therefore the monopolist supplies its full capacity, i.e., $q = k$; the price cap is non-binding since $p = x - k < r + k - k = r$. (Thus, for these realizations changes in the price cap have affects neither the level of output nor the market price.) For high demand realizations, $x > r + k$, the monopolist continues supplying its entire capacity, i.e., $q = k$, but the price cap becomes binding, i.e., $p = r$, and the demand is rationed, i.e., $x - p = x - r > q$. In region B the market price $P(r, k, x)$ depends on the level of capacity.

Table 2C describes the prices and output for $(r, k) \in C$.

X	$[0, 2k)$	$[2k, 1]$
$P(r, k, x)$	$x/2$	$x - k$
$Q(r, k, x)$	$x/2$	k

Table 2C: Equilibrium Output and Price for $(r, k) \in C$.

In region C, the price cap is never binding. The monopolist withholds capacity only for low demand realizations, $x < 2k$, and supplies its entire capacity otherwise. Demand is never rationed. The market price $P(r, k, x)$ depends on the level of capacity.

Note an important feature of equilibrium that stands in contrast to the case where the monopolist is not capacity constrained: when both capacity and the price cap are binding, demand is rationed.

The monopolist's revenue is

$$R(r,k,x) = P(k,r,x)Q(r,k,x),$$

and its expected profit is

$$\bar{\Pi}(r,k) = E\left(R(r,k,X) - ck\right) = E\left(R(r,k,X)\right) - ck.$$

Clearly $\bar{\Pi}$ is continuous on $A \cup B \cup C$.

In equilibrium, the monopolist's capacity maximizes $\bar{\Pi}(r,\cdot)$. Thus, in an interior equilibrium the capacity k^* is such that the monopolist's expected marginal revenue from installing an additional infinitesimal unit of capacity $\overline{MR}(r,k)$, where

$$\overline{MR}(r,k) := \frac{\partial E\left(R(r,k,X)\right)}{\partial k},$$

is equal to the marginal cost of capacity c; i.e., k^* solves the equation

$$\overline{MR}(r,k) = c. \tag{1}$$

In addition, the second order condition

$$\frac{\partial \overline{MR}(r,k)}{\partial k} < 0 \tag{2}$$

holds at k^*.

Using the results described in tables 2A, 2B and 2C we readily calculate the monopolist's expected revenue

$$E\left(R(r,k,X)\right) = \int_0^1 P(k,r,x)Q(r,k,x)f(x)dx$$

for (r,k) in either A, B or C. Differentiating this expression we obtain the expected marginal revenue, which is

$$\overline{MR}(r,k) = \int_{r+k}^1 rf(x)dx = r[1 - F(r+k)] \tag{3}$$

for $(r,k) \in A$,

$$\overline{MR}(r,k) = \int_{2k}^{r+k} \left(x - 2k\right) f(x)dx + \int_{r+k}^1 rf(x)dx \tag{4}$$

for $(r,k) \in B$, and

$$\overline{MR}(r,k) = \int_{2k}^{1} (x-2k) f(x) dx \tag{5}$$

for $(r,k) \in C$. Since (3) and (4) coincide for $k = r$, and (4) and (5) coincide for $r > 1/2$ and $k = 1 - r$, then $\overline{MR}$ in continuous on $A \cup B \cup C$.

In region A, increasing capacity affects the revenue only for high demand realizations $x > r + k$ for which the monopolist supplies its entire capacity. For these demand realizations the price cap r is binding. Thus, the expected revenue increases by r times the probability that the additional marginal unit of capacity is supplied, as shown in equation (3). In region B, a marginal increase of capacity increases revenue not only for high demand realizations $x > r + k$, but also for intermediate demand realizations $2k < x < r + k$, in which the price cap is non-binding and the monopolists supplies its full capacity. In region C, a marginal increase of capacity affects the revenue only when $x > 2k$.

Differentiating $\overline{MR}$ we get

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -rf(r+k) < 0 \tag{6}$$

for $(r,k) \in A$,

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -kf(r+k) - 2\left[F(r+k) - F(2k)\right] < 0 \tag{7}$$

for $(r,k) \in B$, and

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -2\left[1 - F(2k)\right] < 0 \tag{8}$$

for $(r,k) \in C$. Hence the expected marginal revenue function $\overline{MR}$ is decreasing, and therefore the inequality (2) holds on $A \cup B \cup C$. Moreover, since (6) and (7) coincide for $k = r$, then $\overline{MR}$ is differentiable on $A \cup B \cup C$, except perhaps in the boundary of B and C.

Thus, for all $r \in [0,1]$ the monopolist's equilibrium capacity $k^*(r)$ is the unique solution of the equation (1). Moreover, the Maximum Theorem implies that k^* is a continuous function. We summarize these results in Proposition 2.

Proposition 2. *The monopoly equilibrium capacity $k^*(\cdot)$ is a well defined continuous function of the price cap r on $[0,1]$.*

Calculating the equilibrium capacity is somewhat involved. Obviously, the equilibrium capacity is zero for price caps below the unit cost of capacity c. Moreover, it is easy to see that the equilibrium capacity is also zero for price caps r above but near the unit cost of capacity: because the probability of demand realizations $x < c$ is positive, for r above but near c the expected marginal revenue is below c even for $k = 0$. Therefore installing capacity entails losses. Thus, the equilibrium capacity is zero unless the price cap is sufficiently high that expected marginal revenue for levels of capacity near zero is greater than c, i.e., $r \geq \underline{r}(c)$, where $\underline{r}$ is defined by the equation $\overline{MR}(r,0) = c$. Hence, unlike in the setting in which the monopolist makes output decisions ex-post, price caps near the unit cost of capacity are suboptimal.[7]

As in the setting in which the monopolist makes output decisions ex-post, sufficiently large price caps are non-binding. The upper bound on the interval of binding price caps is determined by the distribution of the demand parameter X; specifically this bound $\bar{r}(c)$ is defined by the equation $c = \overline{MR}(r, 1-r)$.

Intermediate price caps $r \in [\underline{r}(c), \bar{r}(c))$ affect the equilibrium capacity in more complex ways. We are able to identify the level of capacity assuming that the hazard rate of X is increasing. In particular, as we shall see in the next section, unlike in the setting in which the monopolist is not capacity constrained, the equilibrium capacity is not monotonically decreasing with the price cap in this interval.

Proposition 3 makes these results precise. Write M^* for the maximum value of $M(r) := \overline{MR}(r,r)$ on $(0, 1/2)$. If $c \in (0, M^*)$, then the equation $M(r) = c$ has two solutions $r_-(c), r_+(c)$, which satisfy $\underline{r}(c) < r_-(c) < r_+(c) < \bar{r}(c) < 1$. If $c \in (M^*, E(X))$, then $c \geq \overline{MR}(r,r)$ for all $r \in [0, 1/2]$. Denote by k_C the solution to the equation

$$\int_{2k}^{1} (x - 2k) f(x) dx = c.$$

(The left hand side of this equation is $\overline{MR}(r,k)$ for $(r,k) \in C$.) Assume that the hazard rate of X is increasing. Then for $r \in (\underline{r}(c), \bar{r}(c))$ the equation

$$\overline{MR}(r,k) = \int_{2k}^{r+k} (x - 2k)\, f(x) dx + \int_{r+k}^{1} r f(x) dx = c.$$

has a unique solution, which we denote by $k_B(r)$. (The left hand side of this equation

[7] If the lower bound of the support of X is $\alpha > c$ (instead of zero as we have assumed), then for $r = c$ the expected marginal revenue is c and profits are zero for $k \in [0, \alpha - c]$, whereas profits are negative for $k > \alpha - c$. Hence the equilibrium capacity may be positive, and may increase or decrease with r near the unit cost of capacity depending of the distribution of demand – see Grimm and Zoettl (2010)'s Section 4 for a discussion of this issue.

is $\overline{MR}(r,k)$ for $(r,k) \in B$.)

Proposition 3. *The equilibrium capacity is determined as follows:*
(3.1) If $r \in [0, \underline{r}(c))$*, then* $k^*(r) = 0$*, and if* $r \in [\bar{r}(c), 1]$*, then* $k^*(r) = k_C$*.*
(3.2) Assume that the hazard rate of X *is increasing. If* $c \in (0, M^*)$*, then*

$$k^*(r) = k_A(r) = F^{-1}(1 - \frac{c}{r}) - r,$$

for $r \in [r_-(c), r_+(c)]$ *and* $k^*(r) = k_B(r)$ *for* $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$*. If* $c \in (M^*, E(X))$*, then* $k^*(r) = k_B(r)$ *for all* $r \in [\underline{r}(c), \bar{r}(c))$*.*

Using the results in tables 2A, 2B and 2C, and the description on the equilibrium capacity given in Proposition 3, we can calculate the expected output and market price as well as the expected (consumer and total) surplus, thus providing a complete description of the monopoly equilibrium. We study in the next section the effect of changes in the price cap on these values.

4 Comparative Statics

In this section we study the comparative static properties of price caps when the hazard rate of X is increasing and its *p.d.f.* f is differentiable. We first show that under these regularity assumptions on the distribution of demand the equilibrium capacity k^* is a *single peaked* function of the price cap r on $(\underline{r}(c), \bar{r}(c))$. Hence the comparative static properties of the equilibrium capacity relative to the price cap that maximizes the equilibrium capacity $r^*(c) \in (\underline{r}(c), \bar{r}(c))$ are analogous to those price caps have on expected output in the setting in which the monopolist is not capacity constrained. We state these results in Proposition 4. The proof of Proposition 4, which is given in Appendix A, establishes these facts by a standard argument that involves implicitly differentiating the equilibrium level of capacity using the first order condition for profit maximization.

Proposition 4. *Assume that the hazard rate of* X *is increasing and its p.d.f.* f *is differentiable. Then* $k^*(\cdot)$ *is a differentiable single peaked function on* $(\underline{r}(c), \bar{r}(c))$*; i.e.,* $k^*(\cdot)$ *has a maximum at some* $r^*(c) \in (\underline{r}(c), \bar{r}(c))$*, and* $dk^*(r)/dr > 0$ *on* $(\underline{r}(c), r^*(c))$ *whereas* $dk^*(r)/dr < 0$ *on* $(r^*(c), \bar{r}(c))$*.*

Next, we discuss the effects of changes in the price cap on the expected output and the expected price. The expected output is readily calculated using the results described in tables 2A, 2B and 2C. In region A, the monopolist maintains idling capacity for intermediate demand realizations in which the price cap is binding. Thus, in region A the expected output strictly decreases with the price cap given the level of capacity. Since for price caps $r \in [r_-(c), r_+(c)]$ the pair $(r, k^*(r)) \in A$, this implies that for price caps in this interval decreasing the price cap increases the expected output provided that the equilibrium capacity does not decrease, i.e.,

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{dE(Q(r, k^*(r), X)}{dr} < 0.$$

Hence when the price cap that maximizes capacity $r^*(c)$ is in the interval $[r_-(c), r_+(c)]$, the expected output decreases with the price cap on $[r^*(c), r_+(c)]$. Therefore the price cap that maximizes output is below $r^*(c)$.

In region B, however, the output does not dependent directly on the price cap, but only indirectly via its impact on the equilibrium level of capacity. Thus, for price caps $r \in [\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, for which $(r, k^*(r)) \in B$, the signs of the effects of changes in the price cap on expected output and capacity are the same, i.e.,

$$\frac{dE(Q(r, k^*(r), X)}{dr} \gtreqless 0 \Leftrightarrow \frac{dk^*}{dr} \gtreqless 0.$$

Let us discuss the effect of changes in the price cap on the expected price. In region A the market price is independent of k, and therefore a change in the price cap only has a direct (positive) effect on P. Hence the expected market price increases with the price cap regardless of its impact on capacity. Since $(r, k^*(r)) \in A$ for $r \in [r_-(c), r_+(c)]$, then

$$\frac{dE(P(r, k^*(r), X)}{dr} > 0$$

on $[r_-(c), r_+(c)]$. In region B, however, the market price depends on k, and therefore a change in the price cap has an indirect effect on the market price via its impact on the level of capacity, as well as a direct (positive) effect. When this indirect effect is also positive, i.e., when $dk^*/dr < 0$, then the total effect is positive, but when the indirect effect is negative, the sign of the total effect is ambiguous. Since $(r, k^*(r)) \in B$ for $r \in [\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{dE(P(r, k^*(r), X)}{dr} > 0,$$

and therefore

$$\frac{dE(P(r,k^*(r),X)}{dr} > 0$$

on $[\underline{r}(c), r^*(c))$. The sign of this derivative on $(r^*(c), \bar{r}(c)]$ is ambiguous. Obviously, changes in the price cap on $[0, \underline{r}(c)) \cup (\bar{r}(c), 1]$ have no effect on the expected price. We summarize these results in Proposition 5, which proof is given in Appendix A.

Proposition 5. *Assume that the hazard rate of X is increasing and its p.d.f. f is differentiable.*
(5.1) If $r^*(c) \in (r_-(c), r_+(c))$, then the expected output decreases with the price cap above and around $r^*(c)$. If $r^*(c) \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then the expected output increases with the price cap on $(\underline{r}(c), r^*(c))$ and decreases on $(r^*(c), \bar{r}(c))$.
(5.2) The expected price increases with the price cap on $[r_-(c), r_+(c)] \cup [r^*(c), \bar{r}(c))$.

Proposition 5 reveals that with demand uncertainty and capacity precommitment the comparative static properties of price caps are complex. In particular, when c is sufficiently small the capacity maximizing price cap $r^*(c)$ does not maximize the expected output: decreasing the price cap below $r^*(c)$ leads to an increase of the expected output even though installed capacity decreases. Of course, this fact has direct implications on the price cap that maximizes the expected surplus, as we shall see in the next section.

5 Optimal Price Caps

A regulator who wants to maximize the expected surplus using a price cap as its single instrument, and cannot force the monopolist to serve its full capacity, must trade off the incentives for capacity investment and capacity withholding, and must account for the cost of installing capacity (some of which may be seldom utilized). Thus, the optimal price cap may differ from the price cap that maximizes capacity investment $r^*(c)$. (In contrast, in the model of full capacity utilization studied by Earle et al. (2007) and Grimm and Zoettl (2010), maximizing the expected surplus simply amounts to maximizing capacity – see Appendix B.) Indeed, we show that when the unit cost of capacity is small this is the case: the optimal price cap is below $r^*(c)$. When the unit cost of capacity is high, however, providing appropriate incentives for capacity investment becomes the dominant objective, and thus the optimal price cap is $r^*(c)$.[8]

Following the literature, we simplify somewhat the problem by assuming efficient rationing, i.e., when demand is rationed the consumers with the largest willingness to pay receive priority to buy the good.

Denote by S the gross surplus (i.e., the surplus ignoring the cost of capacity) as a function of the price cap, capacity and demand realization, (r, k, x). Table 3A describes the function S for (r, k) in region A.

X	$[0, 2r)$	$[2r, r+k)$	$[r+k, 1]$
$S(r, k, x)$	$\frac{3}{8}x^2$	$\frac{1}{2}(x^2 - r^2)$	$\frac{1}{2}(2x - k)k$

Table 3A: Gross Surplus in Region A.

Since for $(r, k) \in A$ the monopolist withholds capacity for demand realizations $x \in [0, r+k)$, then the expected gross surplus depends directly on the price cap, as well as indirectly through its effect on the monopolist capacity decision.

Table 3BC below describes the gross surplus in region $B \cup C$.

X	$[0, 2k)$	$[2k, 1]$
$S(r, k, x)$	$\frac{3}{8}x^2$	$\frac{1}{2}(2x - k)k$

Table 3BC: Gross Surplus in Regions B and C.

When $(r, k) \in B \cup C$ the price cap has no direct effect on the expected gross surplus, but only has an indirect effect via its influence on the monopolist capacity choice.

The (net) surplus is given for $r \in [0, 1]$ by

$$\bar{S}(r) := E(S(r, k^*(r), X) - ck^*(r).$$

An optimal price cap maximizes $\bar{S}$ on $[0, 1]$.

Using the results of tables 3A and 3BC we can readily calculate $\bar{S}$. Differentiating $\bar{S}$ yields

$$\frac{d\bar{S}(r)}{dr} = s(r)I(r)_{[r_-(c), r_+(c)]} + \frac{dk^*(r)}{dr}\left(\int_{r+k^*(r)}^{1}(x - k^*(r))f(x)dx - c\right), \tag{9}$$

[8] Obviously a price cap affects the distribution of surplus also. A regulator who wants to maximize the consumer surplus, for example, would choose as well a price cap below $r^*(c)$ when the cost of capacity is low.

where I is the indicator function, i.e., $I(r)_{[r_-(c),r_+(c)]} = 1$ for $r \in [r_-(c), r_+(c)]$ and $I(r)_{[r_-(c),r_+(c)]} = 0$ otherwise, and

$$s(r) = -r[F(r + k^*(r)) - F(2r)].$$

The expression (9) has a simple interpretation: For price caps $r \in [r_-(c), r_+(c)]$ the price cap-equilibrium capacity pair $(r, k^*(r))$ is in region A. In this region, for intermediate demand realizations the output depends on the price cap. Hence a change in the price cap has a direct effect on surplus captured by the term $s(r)$, as well as an indirect effect via its impact of capacity investment. For price caps $r \in [0,1] \backslash [r_-(c), r_+(c)]$ the price cap-equilibrium capacity pair $(r, k^*(r))$ is in region $B \cup C$. In this region a price cap only has an indirect effect on surplus via its impact of capacity investment. Since the equilibrium capacity is $k^*(r) = r$ in the boundary of regions A and $B \cup C$, then $d\bar{S}(r)/dr$ is continuous, and $\bar{S}$ is differentiable, on $[0,1]$.

If $r^*(c) \in [r_-(c), r_+(c)]$, then $k^*(r) = k_A(r) > r$ and $dk^*(r)/dr \leq 0$ for all $r \in [r_-(c), r^*(c)]$ by Proposition 4, and therefore $s(r) < 0$ and

$$\frac{d\bar{S}(r)}{dr} < 0$$

for all $r \in [r_-(c), r^*(c)]$. Hence the expected surplus decreases with the price cap at $r^*(c)$. Even though decreasing the price cap below $r^*(c)$ decreases capacity, it discourages capacity withholding and increases surplus. Thus, the optimal price cap is below $r^*(c)$.

For $r \in [\underline{r}(c), \bar{r}(c)] \backslash [r_-(c), r_+(c)]$, we show that

$$\frac{d\bar{S}(r)}{dr} = 0 \Leftrightarrow \frac{dk^*(r)}{dr} = 0,$$

and that if $r^*(c) \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then $r^*(c)$ is the unique global maximizer of $\bar{S}$ on $(\underline{r}(c), \bar{r}(c))$ – see the proof of Proposition 6 in Appendix A. Proposition 6 summarizes these results.

Proposition 6. *Assume that hazard rate of X is increasing and its p.d.f. f is differentiable, and let $r^*(c)$ be the capacity maximizing price cap identified in Proposition 4. If $r^*(c) \in [r_-(c), r_+(c)]$ then the expected surplus decreases with the price cap above and around $r^*(c)$, whereas if $r^*(c) \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then $r^*(c)$ maximizes the expected surplus.*

In the absence of capacity precommitment an optimal price cap $r^*(c) = c$ eliminates all inefficiencies. With capacity precommitment, however, an optimal price cap has to trade off the incentives for capacity investment and capacity withholding. We show that a price cap alone is unable to eliminate inefficiencies, i.e., cannot provide the appropriate incentives to install the optimal level of capacity and discourage capacity withholding. Specifically, we show that the level of capacity installed by the monopolist with the optimal price cap, $k^*(r^*(c))$, is below the level that will be socially optimal if the entire capacity was served for each demand realization.

Let us consider the artificial scenario in which a regulator chooses the level of capacity as well as the level of output in order to maximize (net) surplus. In this scenario the surplus realized, denoted by S^*, when the level of capacity is $k \in [0, 1]$ is

$$S^*(k) = \frac{1}{2}\int_0^k x^2 f(x)dx + \frac{1}{2}\int_k^1 (2x - k)kf(x)dx - ck.$$

The socially optimal level of capacity, denoted by k^W, maximizes $S^*(k)$. Differentiating S^* yields

$$\frac{dS^*(k)}{dk} = \int_k^1 (x - k)\, f(x)dx - c,$$

and

$$\frac{d^2S^*(k)}{dk^2} = -[1 - F(k)] < 0.$$

Thus, k^W solves the equation $dS^*(k)/dk = 0$.

It is easy to show that $k^W > k^*(r^*(c))$ – see the proof of Proposition 7 in Appendix A. Thus, a price cap alone cannot provide appropriate incentives to install the optimal level of capacity. In addition, an optimal price cap alone cannot eliminate the inefficiencies arising from capacity withholding.

Proposition 7. *Assume that hazard rate of X is increasing and its p.d.f. f is differentiable. Then the equilibrium capacity with the optimal price cap is below the optimal level of capacity k^W if capacity cannot be withheld. Moreover, the optimal price cap does not eliminate capacity withholding.*

In Appendix B we discuss the impact of price caps when the monopolist cannot withhold capacity, and show by example that in this setting a price cap is not able to induce the monopolist to install the optimal level of capacity either. In this example, in which X is uniformly distributed, the surplus realized and the level of capacity installed with the optimal price cap are below the corresponding surplus and capacity in the setting in which the monopolist may withhold capacity (see Figure 10), which suggests that disallowing capacity withholding may not be an advisable regulatory measure.

6 An Example

Assume that X is uniformly distributed, i.e., $f(x) = 1$. Thus, X has an increasing hazard rate $h(x) = (1-x)^{-1}$, and its *p.d.f.* f is differentiable. Since $E(X) = 1/2$, we consider values of the unit costs of capacity $c \in (0, 1/2)$.

Let us calculate the equilibrium capacity in this setting. The function k_A is given by

$$k_A(r) = F^{-1}(1 - \frac{c}{r}) - r = 1 - \frac{c}{r} - r.$$

The marginal revenue in region B – equation (4) – is

$$\overline{MR}(r,k) = \frac{k^2}{2} + \frac{r}{2}[2(1-2k) - r].$$

Solving equation (1) yields

$$k_B(r) = 2r - \sqrt{2c - r(2-5r)}.$$

The marginal revenue in region C – equation (5) – is

$$\overline{MR}(r,k) = \frac{1}{2}(1-2k)^2.$$

Solving equation (1) yields

$$k_C = \frac{1-\sqrt{2c}}{2}.$$

Let us calculate the functions $\underline{r}$, r_-, r_+ and $\bar{r}$. The function $\underline{r}$ is the solution to the equation

$$c = \overline{MR}(r,0) = \int_0^r xf(x)dx + r(1-F(r)) = \frac{r(2-r)}{2},$$

i.e.,

$$\underline{r}(c) = 1 - \sqrt{1-2c}.$$

The function M is given for $r \in [0,1]$ by

$$M(r) = \overline{MR}(r,r) = r(1-F(2r)) = r(1-2r).$$

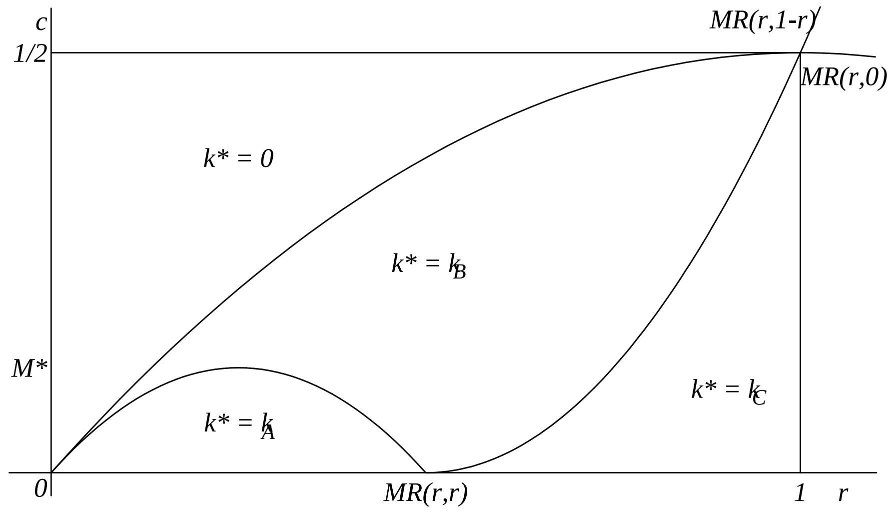

Figure 7: Equilibrium Capacity.

The functions r_- and r_+, i.e., the smaller and larger solutions to the equation $c = M(r)$ are readily calculated as

$$r_-(c) = \frac{1}{4}\left(1 - \sqrt{1-8c}\right), \; r_+(c) = \frac{1}{4}\left(1 + \sqrt{1-8c}\right).$$

These functions are well defined for $c \in (0, 1/8)$, where $M^* = 1/8$ is the maximum value the M. For $c > 1/8$ the above equation has no solution on $[0, 1]$, i.e., the interval $[r_-(c), r_+(c)]$ is empty. The function $\bar{r}$ solves the equation

$$c = \overline{MR}(r, 1-r) = \int_{2(1-r)}^{1} xf(x)dx - 2(1-r)\left[1 - F(2(1-r))\right] = \frac{(1-2r)^2}{2},$$

i.e.,

$$\bar{r}(c) = \frac{1+\sqrt{2c}}{2}.$$

It is easy to check that

$$c < \underline{r}(c) < \frac{1}{2} < \bar{r}(c) < 1$$

for $c \in (0, 1/2)$, and

$$\underline{r}(c) < r_-(c) < r_+(c) < \frac{1}{2} < \bar{r}(c)$$

for $c \in (0, 1/8)$.

Figure 7 provides a description of the function $k^*(r)$ for $c \in (0, 1/2)$. For $c \leq 1/9$ the equilibrium capacity $k^*(r)$ reaches its maximum at the price cap $r_A^* = \sqrt{c} \in [r_-(c), r_+(c)]$. For $c > 1/9$, the equilibrium capacity $k^*(r)$ reaches its maximum at

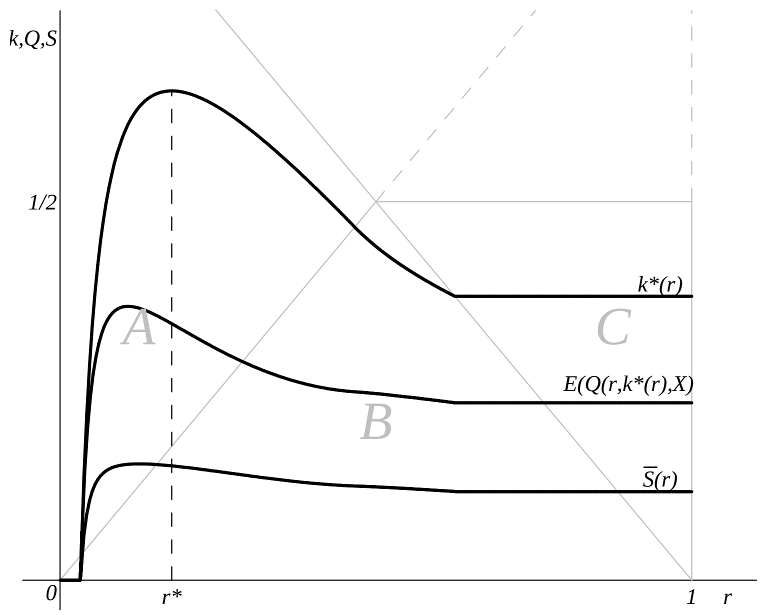

Figure 8: Capacity, Expected Output, and Surplus for $c = 1/32$.

$r_B^* = \left(1 + 2\sqrt{10c - 1}\right)/5 \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. Interestingly, for $c \in (1/9, 1/8)$ the equilibrium capacity $k^*(r)$ is increasing in the interval $(r_-(c), r_+(c)]$, and reaches its maximum at $r^*(c) \in (r_+(c), \bar{r}(c))$.

We calculate the expected surplus. If $r < \underline{r}(c)$, then the expected surplus is $\bar{S}(r) = 0$. If $r \in [r_-(c), r_+(c)]$, which requires $c < 1/8$, then the expected surplus is

$$\bar{S}(r) = \frac{r^3 \left(1 + 4r^3\right) + 3r^2 \left(c \left(c - 2r \left(1 - r\right)\right) - r^3\right) - c^3}{6r^3}.$$

If $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then the expected surplus is

$$\bar{S}(r) = \frac{r}{2} \left(4 - 9r\right) - c(1 + 2r) + \left(c + 2r - \frac{1}{2}\right) \sqrt{2c - r \left(2 - 5r\right)}.$$

And if $r \in [\bar{r}(c), 1]$ then

$$\bar{S}(r) = \frac{1 - 6c}{8} + \frac{\sqrt{2c^3}}{2}.$$

Figure 8 displays the equilibrium capacity and surplus as functions of the price cap when the unit cost of capacity is $c = 1/32$. The price cap that maximizes capacity is $r_A^* = \sqrt{2}/8$ whereas, consistently with Proposition 6, the expected surplus is maximized at $r = 1/8 < r_A^*$.

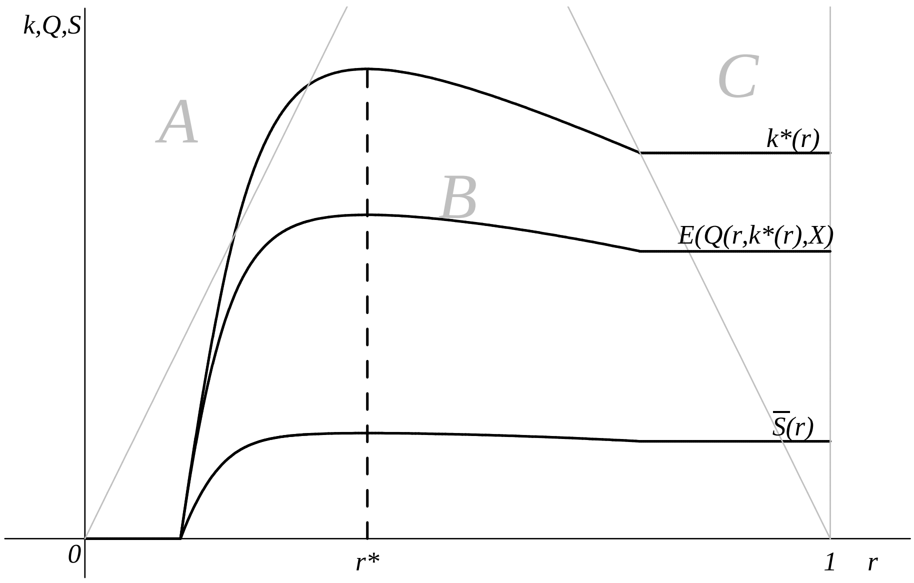

Figure 9: Capacity, Expected Output, and Surplus for $c = 3/25$.

Figure 9 shows the graphs of the capacity and the expected surplus for $c = 3/25$. For this unit cost of capacity we have $[r_-(c), r_+(c)] = [2/10, 3/10]$. (Note that $c = 3/25 < 1/8$.) The price cap that maximizes both capacity and expected surplus is $r_B^* = (2\sqrt{5}+5)/25 \in (\underline{r}(c), \bar{r}(c))$, i.e., the maximum capacity is reached at a price cap-capacity pair in region B, and consistently with Proposition 6, the expected surplus is maximal at this price cap.

Suppose that a regulator chooses the level of capacity, assuming that for each demand realization the entire capacity is served to the consumers that value the good the most, in order to maximize surplus. Using the results obtained in Section 5 we calculate the expected surplus as a function of the capacity as

$$S^*(k) = \frac{k^2\,(k-3)}{6} + \frac{k\,(1-2c)}{2},$$

which is maximized at $k^W = 1 - \sqrt{2c}$.

For $c = 1/32$ the optimal price cap is $r^W < r^*$, the level of capacity installed is $k^*(r^W) = (0.86)\,k^W$, and the expected surplus is $\bar{S}(r^W) = (0.93) S^*(k^W)$. For $c = 3/25$, the optimal price cap is $r^W = r^*$, and $k^*(r^W) \simeq (0.61) k^W$ and $\bar{S}(r^W) = (0.81) S^*(k^W)$. These numbers suggest that with capacity withholding price caps are more effective when unit cost of capacity is small than when it is large.

7 Conclusions

In the absence of quantity precommitment, whether the demand is deterministic or stochastic, price cap regulation provides an effective instrument to mitigate market power: If firms produce the good with constant returns to scale, for example, decreasing the price cap (while maintaining it above marginal cost) leads to an increase of (expected) output and surplus, and to a decrease of the market price. Moreover, a price cap equal to marginal cost is able to eliminate inefficiencies.

With demand uncertainty and capacity precommitment (and withholding), price cap regulation has to deal with a trade off involving the incentives for capacity investment and capacity withholding: decreasing the price cap alleviates capacity withholding but may discourage capacity investment. As a consequence, the optimal price cap may not maximize capacity investment. Indeed, when the cost of capacity is low maximizing the expected surplus calls for a low price cap in order to discourage capacity withholding, even at the cost of reducing capacity investment. Moreover, under standard regularity assumptions on the demand, the comparative static properties of price caps above the price cap that maximizes capacity are analogous to those obtained in the case of a deterministic demand. Price cap regulation provides an useful instrument to mitigate market power and enhance efficiency, although it cannot restore efficiency.

It is noteworthy that even if capacity withholding is not an issue, i.e., even if the regulator may enforce full capacity utilization, price cap regulation does not provide appropriate incentives for capacity investment either. In fact, as the example discussed in Appendix B shows, both capacity investment and surplus may be smaller with full capacity utilization than with capacity withholding.

8 Appendix A: Proofs

Proof of Proposition 3. Assume that the hazard rate of X, $h(\cdot) = f(\cdot)/[1-F(\cdot)]$, is increasing. We calculate the equilibrium capacity $k^*(r)$. Let us consider first price caps $r \in [0, 1/2]$. Then $\bar{\Pi}(r, \cdot)$ takes values in regions A and B.

If the capacity that maximizes $\bar{\Pi}(r, \cdot)$ is such that $(r, k) \in A$, then solving the equation (1) for $\overline{MR}$ given by (6) yields

$$k_A(r) = F^{-1}(1 - \frac{c}{r}) - r.$$

Hence

$$k_A(r) + r = F^{-1}(1 - \frac{c}{r}) < 1,$$

and therefore $k_A(r) < 1 - r$. If $(r, k_A(r)) \in A$, then $r \leq k_A(r)$. This inequality is equivalent to

$$c \leq r\,(1 - F(2r)) = \overline{MR}(r, r).$$

Write $M(r) := \overline{MR}(r, r)$. Differentiating M yields

$$\frac{dM(r)}{dr} = (1 - F(2r)) - 2rf(2r) = (1 - F(2r))\,(1 - 2rh(2r))\,,$$

which is positive for values of r close to zero and negative for values of r close to $1/2$. Since h is increasing, then the function $M(r)$ is strictly concave and reaches its maximum value M^* on $(0, 1/2)$. If $c < M^*$, then the equation $\overline{MR}(r, r) = c$ has two solutions on $(0, 1/2)$, which we denote by $r_-(c)$ and $r_+(c)$ with $r_-(c) < r_+(c)$. In this case, for $r \in [r_-(c), r_+(c)]$, we have $(r, k_A^*(r)) \in A$. If $r \notin [r_-(c), r_+(c)]$, i.e., $c > \overline{MR}(r, r)$, then $\bar{\Pi}(r, \cdot)$ decreases with k in region A, and reaches its maximum in region B.

Assume that the capacity that maximizes $\bar{\Pi}(r, \cdot)$ is such that $(r, k) \in B$. Denote by $k_B(r)$ the solution to equation (1) for $\overline{MR}$ given by (4). Hence $k_B(r)$ satisfies

$$0 < k_B(r) < r.$$

(Recall that we are identifying the monopolist capacity for $r < 1/2$, and therefore $k_B(r) < r$ implies $k_B(r) < 1 - r$.) The inequality $k_B(r) < r$ is equivalent to

$$c > \overline{MR}(r, r).$$

If $c \leq \overline{MR}(r, r)$, i.e., $r \in [r_-(c), r_+(c)]$, then $\bar{\Pi}(r, \cdot)$ increases with k in region B, and reaches its maximum in region A. The inequality $k_B(r) > 0$ is equivalent to

$$c < \int_0^r x f(x) dx + r\,(1 - F(r)) = \overline{MR}(r, 0),$$

i.e., the expected marginal revenue when output is zero $\overline{MR}(r, 0)$ must be greater than the unit cost of capacity c. If this inequality does not hold, then $\bar{\Pi}(r, \cdot)$ decreases with

k in region B and reaches its maximum at $k^* = 0$. Since $d\overline{MR}(r,0)/dr = 1 - F(r) > 0$ on $(0,1)$, then the function $\overline{MR}(\cdot, 0)$ has an inverse, which we denote by $\underline{r}$. Then the condition $c < \overline{MR}(r,0)$ may be written as $r > \underline{r}(c)$. Since

$$\overline{MR}(r,0) < \int_0^r xf(x)dx + r\,(1 - F(r)) = r,$$

then

$$c = \overline{MR}(\underline{r}(c), 0) < \underline{r}(c).$$

Therefore the equilibrium capacity is $k^* = 0$ for a range of price caps above the cost of capacity, $r \in (c, \underline{r}(c)]$. Also, since

$$\overline{MR}(r,0) > r\,(1 - F(r)) > r\,(1 - F(2r)) = \overline{MR}(r,r),$$

then $r < \underline{r}(c)$ (i.e., $c > \overline{MR}(r,0)$) implies $r < r_-(c)$.

Let us now consider price caps $r \in (1/2, 1]$. Then $\bar{\Pi}(r,\cdot)$ takes values in regions B and C.

Assume that the capacity that maximizes $\bar{\Pi}(r,\cdot)$ is such that $(r,k) \in B$. If $r \leq \underline{r}(c)$, then $\bar{\Pi}(r,\cdot)$ decreases with k and reaches its maximum at $k = 0$. If $r > \underline{r}(c)$, then $\bar{\Pi}(r,\cdot)$ reaches its maximum in region B if the solution to condition (1), $k_B(r)$, satisfies

$$k_B(r) < 1 - r.$$

This condition is equivalent to

$$c > \int_{2(1-r)}^1 xf(x)dx - 2\,(1-r)\,[1 - F(2\,(1-r))] = \overline{MR}(r, 1-r).$$

Note that

$$\frac{d\overline{MR}(r,1-r)}{dr} = 2(1 - F(2\,(1-r))) > 0.$$

Hence the function $\overline{MR}(r, 1-r)$ has an inverse on $(1/2, 1)$, which we denote by $\bar{r}(c)$, and therefore we may write the above inequality as $r < \bar{r}(c)$. If $r \geq \bar{r}(c)$, then $\bar{\Pi}(r,\cdot)$ increases with k in region B and reaches its maximum in region C. Note that for $r = 1$ we have $\overline{MR}(r, 1-r) = \overline{MR}(1,0) = E(X)$. Hence, since $c < E(X)$ by assumption, we have $\bar{r}(c) < 1$.

Finally, assume that the capacity that maximizes $\bar{\Pi}(r,\cdot)$ is such that $(r,k) \in C$. Denote by k_C the solution to the condition (1) for $\overline{MR}$ given by equation (5). Clearly k_C is independent of the price cap r. Also, since $\overline{MR}(r, 1/2) = 0$, then $k_C < 1/2$

for all $c \in (0, E(X))$. Since the expected marginal revenue decreases with k, then $k_C > 1-r$ implies $c < \overline{MR}(r, 1-r)$. Moreover, since $r > 1/2$ and $\overline{MR}$ is decreasing, then $\overline{MR}(r, 1-r) < \overline{MR}(r, r)$. Hence k_C solves the monopolist problem if $r \geq \bar{r}(c)$. Otherwise, i.e., if $r < \bar{r}(c)$, then $\bar{\Pi}(r, \cdot)$ decreases with k in region C and reaches its maximum in region B.

As shown above $c < \underline{r}(c)$. If $c < M^*$, then we have $\underline{r}(c) < r_-(c) < r_+(c) < 1/2$. Since $1/2 < \bar{r}(c) < 1$, then

$$c < \underline{r}(c) < r_-(c) < r_+(c) < 1/2 < \bar{r}(c) < 1.$$

If $c \geq M^*$, then $c \geq \overline{MR}(r, r)$ for all $r \in [0, 1/2]$, and the equilibrium capacity lies in region B for all $r \in [0, 1/2]$. □

The following lemma will be useful in the proof of Proposition 4.

Lemma 1. *Let g be a real valued function on $\mathbb{R}$, differentiable on some interval (a, b), and satisfying $g'(a) > 0 > g'(b)$, and $g''(y) < 0$ for all $y \in (a, b)$ such that $g'(y) = 0$. Then g has a unique global maximizer on $[a, b]$, $y^* \in (a, b)$, and g' is positive on (a, y^*) and negative on (y^*, b).*

Proof. Let $y^* = \sup\{y \in (a, b) \mid g'(y) > 0\}$ and $y^{**} = \inf\{y \in (a, b) \mid g'(y) < 0\}$. Since g' is continuous on (a, b), then $g'(y^*) = g'(y^{**}) = 0$, and therefore $a < y^{**} \leq y^* < b$. We show that $y^* = y^{**}$, which establishes the lemma. Suppose by way of contradiction that $y^{**} < y^*$. Since both $g''(y^*)$ and $g''(y^{**})$ are negative, then for $\varepsilon \in (0, y^* - y^{**})$ sufficiently small

$$g'(y^{**} + \varepsilon) < 0 < g'(y^* - \varepsilon).$$

Hence $g'(\bar{y}) = 0$ for some $\bar{y} \in (y^{**} - \varepsilon, y^* + \varepsilon)$, and g' is negative (positive) for y below (above) and near $\bar{y}$. Hence $g''(\bar{y}) > 0$, which is a contradiction. □

Proof of Proposition 4. Let $r \in (\underline{r}(c), \bar{r}(c))$. Since the expected marginal revenue $\overline{MR}(r, k)$ is differentiable in regions $A \cup B$, we can differentiate equation (1) to get

$$\frac{\partial \overline{MR}(r, k)}{\partial k} dk + \frac{\partial \overline{MR}(r, k)}{\partial r} dr = 0.$$

And since $\overline{MR}$ is decreasing, i.e.,

$$\frac{\partial \overline{MR}(r, k)}{\partial k} < 0,$$

then

$$\frac{dk^*}{dr} = -\frac{\partial \overline{MR}(r,k)}{\partial r}\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-1},$$

and

$$\frac{dk^*}{dr} \gtreqqless 0 \Leftrightarrow \frac{\partial \overline{MR}(r,k)}{\partial r} \gtreqqless 0.$$

Since f is differentiable, then $\overline{MR}$ is twice differentiable, and

$$\begin{aligned}
\frac{d^2k^*}{dr^2} &= -\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-1} \frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) \\
&\quad + \frac{\partial \overline{MR}(r,k)}{\partial r}\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-2} \frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial k}\right) \\
&= -\left(\frac{\partial \overline{MR}(r,k)}{\partial k}\right)^{-1}\left(\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) + \frac{dk^*}{dr}\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial k}\right)\right).
\end{aligned}$$

Hence, for r such that $dk^*/dr = 0$, we have

$$\frac{d^2k^*}{dr^2} \gtreqqless 0 \Leftrightarrow \frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) \gtreqqless 0.$$

If $(r, k^*(r)) \in A$, then differentiating $\overline{MR}$ given in (3) yields

$$\frac{\partial \overline{MR}(r,k)}{\partial r} = 1 - F(r+k) - rf(r+k) = (1 - F(r+k))\left(1 - rh\left(r+k\right)\right),$$

and

$$\begin{aligned}
\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) &= -f(r+k)\left(1 + \frac{dk_A}{dr}\right)\left(1 - rh\left(r+k\right)\right) \\
&\quad - \left(1 - F(r+k)\right)\left(h\left(r+k\right) + rh'\left(r+k\right)\right)\left(1 + \frac{dk_A}{dr}\right).
\end{aligned}$$

Assume that $dk_A/dr = 0$. Then $1 - rh\left(r + k^*(r)\right) = 0$, and

$$\frac{d}{dr}\left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r}\right) = -\left(1 - F(r+k^*(r))\right)\left(h\left(r+k^*(r)\right) + rh'\left(r+k^*(r)\right)\right).$$

If the hazard rate is increasing (i.e., $h' > 0$), then we have

$$\frac{d^2k_A}{dr^2} < 0,$$

and therefore every critical point of k_A is a local maximum.

If $(r, k_B(r)) \in B$, then differentiating $\overline{MR}$ given in (4) yields

$$\frac{\partial \overline{MR}(r,k)}{\partial r} = 1 - F(r+k) - kf(r+k) = (1 - F(r+k))\left(1 - kh(r+k)\right),$$

and

$$\begin{aligned}
\frac{d}{dr}\left(\frac{\partial\overline{MR}(r,k^*(r))}{\partial r}\right) &= -f(r+k^*(r))\left(1-k^*(r)h(r+k^*(r))\right)\left(1+\frac{dk_B}{dr}\right) \\
&\quad -\left(1-F(r+k^*(r))\right)k^*(r)h'(r+k^*(r))\left(1+\frac{dk_B}{dr}\right) \\
&\quad -\left(1-F(r+k^*(r))\right)h(r+k^*(r))\frac{dk_B}{dr}.
\end{aligned}$$

Assume that $dk_B/dr = 0$. Then $1-k^*(r)h\left(r+k^*(r)\right)=0$, and

$$\frac{d}{dr}\left(\frac{\partial\overline{MR}(r,k^*(r))}{\partial r}\right) = -\left(1-F(r+k^*(r))\right)k^*(r)h'(r+k^*(r)).$$

If the hazard rate is increasing (i.e., $h'>0$) we have

$$\frac{d^2k_B}{dr^2}<0,$$

and therefore every critical point of k_B is a local maximum.

Thus, for $r\in(\underline{r}(c),\bar{r}(c))$, $d^2k^*(r)/dr^2<0$ whenever $dk^*(r)/dr=0$. Moreover, since $k_B(\bar{r}(c))=1-\bar{r}(c)$, and

$$\begin{aligned}
\left.\frac{\partial\overline{MR}(r,1-r)}{\partial r}\right|_{r=\bar{r}(c)} &= 1-F\left(\bar{r}(c)+(1-\bar{r}(c))\right)-(1-\bar{r}(c))\,f\left(\bar{r}(c)+(1-\bar{r}(c))\right) \\
&= -\left(1-\bar{r}(c)\right)f(1) \\
&< 0,
\end{aligned}$$

then $dk_B(\bar{r}(c))/dr<0$. And since $k_B(\underline{r}(c))=0$, and

$$\left.\frac{\partial\overline{MR}(r,0)}{\partial r}\right|_{r=\underline{r}(c)} = 1-F(\underline{r}(c))>0,$$

then $dk_B(\underline{r}(c))/dr>0$. Hence k^* has a global maximum at some $r^*(c)\in(\underline{r}(c),\bar{r}(c))$, and satisfies $dk^*/dr>0$ on $(\underline{r}(c),r^*(c))$ and $dk^*/dr<0$ on $(r^*(c),\bar{r}(c))$ by Lemma 1. Since k^* is continuous on $[0,1]$, is equal to zero on $[0,\underline{r}(c)))$ and is equal to k_C on $[\bar{r}(c),1)$, this implies that k^* is quasi-concave, i.e., single peak, on $[0,1]$. $\square$

Proof of Proposition 5. The expected output is

$$E(Q(r,k^*(r),X)=\int_0^{2r}\frac{x}{2}f(x)dx+\int_{2r}^{r+k^*(r)}(x-r)f(x)dx+\int_{r+k^*(r)}^1 k^*(r)f(x)dx,$$

for $r\in[r_-(c),r_+(c)]$, and

$$E(Q(r,k^*(r),X)=\int_0^{2k^*(r)}\frac{x}{2}f(x)dx+\int_{2k^*(r)}^1 k^*(r)f(x)dx$$

for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. Hence

$$\frac{dE(Q(r, k^*(r), X)}{dr} = -[F(r + k^*(r)) - F(2r)] + \frac{dk^*}{dr}(1 - F(r + k^*(r)))$$

for $r \in [r_-(c), r_+(c)]$, and

$$\frac{dE(Q(r, k^*(r), X)}{dr} = \frac{dk^*}{dr}(1 - F(2k^*(r)))$$

for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$.

Thus,

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{dE(Q(r, k^*(r), X)}{dr} < 0$$

for $r \in [r_-(c), r_+(c)]$, that is, the expected output decreases with the price cap beyond the price cap that maximizes capacity, and therefore the price cap that maximizes output is below $r^*(c)$. Moreover,

$$\frac{dE(Q(r, k^*(r), X)}{dr} \gtreqless 0 \Leftrightarrow \frac{dk^*}{dr} \gtreqless 0.$$

for $r \in [\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, that is, the expected output increases with the price cap for $r \in (\underline{r}(c), r^*(c))$, and decreases for $r \in (r^*(c), \bar{r}(c))$.

Likewise for $r \in [r_-(c), r_+(c)]$ the expected price is

$$E(P(r, k^*(r), X) = \int_0^{2r} \frac{x}{2} f(x)dx + \int_{2r}^1 rf(x)dx,$$

and for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$ it is

$$E(P(r, k^*(r), X) = \int_0^{2k^*(r)} \frac{x}{2} f(x)dx + \int_{2k^*(r)}^{r+k^*(r)} (x - k^*(r))f(x)dx + \int_{r+k^*(r)}^1 rf(x)dx.$$

Hence, for $r \in [r_-(c), r_+(c)]$

$$\frac{dE(P(r, k^*(r), X)}{dr} = 1 - F(2r) > 0.$$

Also, for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$,

$$\frac{dE(P(r, k^*(r), X)}{dr} = -\frac{dk^*}{dr}[F(r + k^*(r)) - F(2k^*(r))] + [1 - F(r + k^*(r))],$$

and therefore

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{dE(P(r, k^*(r), X)}{dr} > 0. \ \square$$

Proof of Proposition 6. The expected gross surplus is

$$\begin{aligned} E(S(r,k,X)) &= \frac{3}{8}\int_0^{2r} x^2 f(x)dx + \frac{1}{2}\int_{2r}^{r+k}(x^2-r^2)f(x)dx \\ &\quad +\frac{1}{2}\int_{r+k}^{1}(2x-k)kf(x)dx. \end{aligned} \tag{10}$$

for $(r,k) \in A$, and is

$$E(S(r,k,X)) = \frac{3}{8}\int_0^{2k} x^2 f(x)dx + \frac{1}{2}\int_{2k}^{1}(2x-k)\,kf(x)dx. \tag{11}$$

for $(r,k) \in B \cup C$.

For $r \in [0,1]$ the net surplus is $\bar{S}(r) = E(S(r,k^*(r),X)) - ck^*(r)$.

For price caps $r \in [r_-(c), r_+(c)]$ the price cap-equilibrium capacity pair $(r, k^*(r))$ is in region A. Differentiating $\bar{S}$ given in (10) yields

$$\frac{d\bar{S}(r)}{dr} = -r[F(r+k^*(r)) - F(2r)] + \frac{dk^*(r)}{dr}\left(\int_{r+k^*(r)}^{1}(x-k^*(r))f(x)dx - c\right),$$

Recall that $r^*(c)$ is the capacity maximizing price cap identified in Proposition 4. If $r^*(c) \in [r_-(c), r_+(c)]$, then $dk^*(r^*(c))/dr = 0$ and $k^*(r^*(c)) = k_A(r^*(c)) > r^*(c)$ imply

$$\frac{d\bar{S}(r^*(c))}{dr} = -r^*(c)[F(r^*(c)+k^*(r^*(c))) - F(2r^*(c))] < 0.$$

Hence the optimal price cap is below $r^*(c)$.

For $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$ we have $(r, k^*(r)) \in B \cup C$. Differentiating $\bar{S}$ given in (11) yields

$$\frac{d\bar{S}(r)}{dr} = \frac{dk^*(r)}{dr}\left(\int_{2k^*(r)}^{1}(x-k^*(r))f(x)dx - c\right).$$

Since $(r, k^*(r)) \in B$, then $k^*(r) < r$, and

$$\overline{MR}(r,k^*(r)) = \int_{2k^*(r)}^{r+k^*(r)}(x-2k^*(r))\,f(x)dx + \int_{r+k^*(r)}^{1} rf(x)dx = c.$$

Hence

$$\int_{2k^*(r)}^{1}(x-k^*(r))f(x)dx - c = \int_{2k^*(r)}^{r+k^*(r)} k^*(r)f(x)dx + \int_{r+k^*(r)}^{1}(x-k^*(r)-r)f(x)dx > 0,$$

and therefore

$$\frac{d\bar{S}(r)}{dr} = 0 \Leftrightarrow \frac{dk^*(r)}{dr} = 0.$$

Differentiating $d\bar{S}(r)/dr$ we get

$$\begin{aligned}\frac{d^2\bar{S}(r)}{dr^2} &= \frac{d^2k^*(r)}{dr^2}\left(\int_{2k^*(r)}^{1}(x-k^*(r))f(x)dx-c\right)\\ &\quad -\left(\frac{dk^*(r)}{dr}\right)^2[1-F(2k^*(r))]-2k^*(r)f(2k^*(r)).\end{aligned}$$

If $d\bar{S}(r)/dr=0$, then $dk^*(r)/dr=0$, which as shown above implies $d^2k^*(r)/dr^2<0$. Hence $d^2\bar{S}(r)/dr^2<0$. Thus, by Lemma 1 if $r^*(c)\in(\underline{r}(c),\bar{r}(c))\backslash[r_-(c),r_+(c)]$, then $r^*(c)$ is the unique global maximizer of $\bar{S}$ on $(\underline{r}(c),\bar{r}(c))$. $\square$

Proof of Proposition 7. We show that $k^W>k^*(r^*(c))\geq k^*(r)$ for all $r\in[0,1]$. Let us fix c and reduce notation by writing k^* and r^* for $k^*(r^*(c))$ and $r^*(c)$, respectively. If $r^*\in[r_-(c),r_+(c)]$, then $k^*\geq r^*$ and

$$\overline{MR}(r^*,k^*)=\int_{r^*+k^*}^{1}r^*f(x)dx=c$$

imply

$$\begin{aligned}\left.\frac{dS^*(k)}{dk}\right|_{k=k^*} &= \int_{k^*}^{1}(x-k^*)\,f(x)dx-\int_{r^*+k^*}^{1}r^*f(x)dx\\ &= \int_{k^*}^{r^*+k^*}(x-k^*)f(x)dx+\int_{r^*+k^*}^{1}(x-r^*-k^*)f(x)dx\\ &> 0.\end{aligned}$$

Hence $k^W>k^*$. If $r^*\in(\underline{r}(c),\bar{r}(c))\backslash[r_-(c),r_+(c)]$, then $k^*\leq r^*$ and

$$\overline{MR}(r^*,k^*)=\int_{2k^*}^{r^*+k^*}(x-2k^*)f(x)dx+\int_{r^*+k^*}^{1}r^*f(x)dx=c$$

imply

$$\begin{aligned}\left.\frac{dS^*(k)}{dk}\right|_{k=k^*} &= \int_{k^*}^{1}(x-k^*)f(x)dx-\left(\int_{2k^*}^{r^*+k^*}(x-2k^*)f(x)dx+\int_{r^*+k^*}^{1}rf(x)dx\right)\\ &= \int_{k^*}^{2k^*}(x-k^*)\,f(x)dx\\ &\quad +\int_{2k^*}^{r^*+k^*}k^*f(x)dx+\int_{r^*+k^*}^{1}(x-r^*-k^*)f(x)dx\\ &> 0.\end{aligned}$$

Hence $k^W>k^*$ as well. $\square$

9 Appendix B: Full Capacity Utilization

Let us consider an alternative model in which the monopolist chooses its output before the demand is realized and supplies it unconditionally. This setting is similar to that of Section 2, except that output decision is made ex-ante. Alternatively, renaming output as capacity we may view this model as a variation of the model discussed in sections 3 to 6, except that the monopolist cannot withhold capacity, i.e., must supply its entire capacity for each demand realization, and therefore its only choice is how much capacity to install. This model is studied by Earle et al. (2007) and Grim and Zoettl (2010). In order to discuss the differing effects of price caps in this setting and in the setting in which the monopolist may withhold capacity, we describe the monopolist's expected profit and the expected surplus, and then calculate the equilibrium and study the comparative static properties of price caps assuming that X is distributing uniformly.

Monopoly Equilibrium without Capacity Withholding

Assume that a regulatory agency imposes a price cap $r \in [0, 1]$. Table B1 identifies the market equilibrium price, denoted by $\hat{P}$, when the price cap is binding, i.e., $k < 1 - r$.

X	$[0, k)$	$[k, r + k)$	$[r + k, 1]$
$\hat{P}(r, k, x)$	0	$x - k$	r

Table B1: Equilibrium Price for $k \in [0, 1 - r)$.

Table B2 identifies the market equilibrium price when the price cap is not binding, i.e., $k > 1 - r$.

X	$[0, k)$	$[k, 1]$
$\hat{P}(r, k, x)$	0	$x - k$

Table B2: Equilibrium Price for $k \in [1 - r, 1]$.

Hence the expected price is

$$E(\hat{P}(r, k, X)) = \int_{k}^{r+k} (x - k)\, f(x)dx + \int_{r+k}^{1} rf(x)dx$$

for $k < 1 - r$, and

$$E(\hat{P}(r, k, X)) = \int_{k}^{1} (x - k) f(x)dx$$

for $k \geq 1 - r$. The monopolist chooses the level of capacity in order to maximize its expected profit $\hat{\Pi}$ given for $(r, k) \in [0, 1]$ by

$$\hat{\Pi}(r, k) = E\left([\hat{P}(r, k, X) - c]k\right) = [E(\hat{P}(r, k, X)) - c]k.$$

Clearly $\hat{\Pi}$ is continuous on $[0, 1]^2$.

Let $r \in [0, 1]$. In an interior equilibrium k solves $\partial\hat{\Pi}(r, k)/\partial k = 0$ and satisfies $\partial^2\hat{\Pi}(r, k)/\partial k^2 \leq 0$. We have

$$\frac{\partial^2\hat{\Pi}(r, k)}{\partial k^2} = -k(f(r + k) - f(k)) - 2(F(r + k) - F(k))$$

for $k < 1 - r$, and

$$\frac{\partial^2\hat{\Pi}(r, k)}{\partial k^2} = kf(k) - 2(1 - F(k))$$

for $k \geq 1-r$. The signs of these expressions are ambiguous, and therefore it is unclear whether the monopolist's expected profit $\hat{\Pi}(r, \cdot)$ is a concave function. (In fact, it is not difficult to find examples for which $\hat{\Pi}(r, \cdot)$ is not concave – e.g., take $f(x) = 2(1 - x)$ and $r = 1/4$.) Thus, in contrast to the setting in which the monopolist may withhold capacity, in the present setting uniqueness of equilibrium is not warranted.

Table B3 below describes the gross surplus, denoted by $\hat{S}$, assuming efficient rationing. Note that $\hat{S}$ is independent of r.

X	$[0, k)$	$[k, 1)$
$\hat{S}(r, k, x)$	$\frac{1}{2}x^2$	$\frac{1}{2}k(2x - k)$

Table B3: Surplus.

Assume that for each $r \in [0, 1]$ there is a unique monopoly equilibrium, which we denote by $\hat{k}^*(r)$. Then for each r the expected (net) surplus is

$$E(\hat{S}(r, \hat{k}^*(r), X)) - c\hat{k}^*(r).$$

Since $\hat{S}$ is independent of r, and since profit maximization implies $E(\hat{P}(r, \hat{k}^*(c), X)) > c$ whenever $\hat{k}^*(c) > 0$, then maximizing the expected surplus amounts to maximizing capacity, i.e., the optimal price cap maximizes the level of installed capacity. For each $c \in (0, E(X))$ denote by $\hat{r}^*(c)$ the price cap maximizes capacity, and

$$\hat{S}^*(c) := E(\hat{S}(r, \hat{k}^*(\hat{r}^*(c)), X) - c\hat{k}^*(\hat{r}^*(c))$$

for the maximum expected surplus.

An Example: The Uniform Distribution

Assume that X is uniformly distributed and $c \in (0, 1/2)$. Hence

$$E(\hat{P}(r,k,X)) = \frac{1}{2} r\,(2 - 2k - r)$$

for $k < 1 - r$, and

$$E(\hat{P}(r,k,X)) = \frac{1}{2}\,(1-k)^2$$

for $k \geq 1 - r$. Moreover,

$$\frac{\partial^2 \hat{\Pi}(r,k)}{\partial k^2} = -2r$$

for $k < 1 - r$, and

$$\frac{\partial^2 \hat{\Pi}(r,k)}{\partial k^2} = -2 + 3k$$

for $k \geq 1 - r$. Therefore $\hat{\Pi}(r, \cdot)$ is strictly concave, and the monopoly equilibrium $\hat{k}^*(r)$ is unique, for all $r \in (0, 1]$. If the equilibrium capacity is $k < 1 - r$, then the equilibrium capacity solves the equation

$$\frac{\partial \hat{\Pi}(r,k)}{\partial k} = -rk + \frac{1}{2} r\,(2 - 2k - r) - c = 0.$$

Solving this equation we get

$$k_1(r) = \frac{1}{2}\left(1 - \frac{c}{r} - \frac{r}{2}\right).$$

Hence $k_1(r)$ is the solution to the monopolist problem provided $0 < k_1(r) < 1 - r$, i.e.,

$$\underline{r}(c) := 1 - \sqrt{1-2c} < r < \frac{1}{3}\sqrt{6c+1} + \frac{1}{3} := \overline{r}(c).$$

If $r < \underline{r}(c)$, then expected profit decreases with k and the equilibrium capacity is $k^* = 0$. If $r > \overline{r}(r)$, then expected profit increases with k at $k = 1 - r$.

If the equilibrium capacity is $k \geq 1 - r$, then the equilibrium capacity solves the equation

$$-(1-k)\,k + \frac{1}{2}\,(1-k)^2 = c.$$

Solving this equation we get

$$k_2 = \frac{2 - \sqrt{1+6c}}{3}.$$

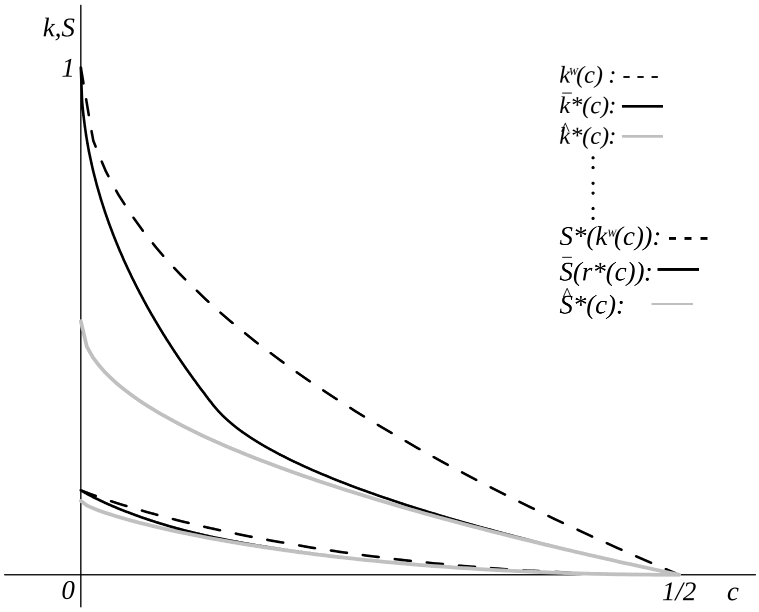

Figure 10: Capacity Investment and Surplus with and without Withholding.

Note that $k_2 > 0$ for all $c \in (0, 1/2)$. Hence k_2 is the solution to the monopolist problem provided $k_2 \geq 1 - r$, i.e., $r \geq \overline{r}(c)$. If $r < \overline{r}(c)$ the expected profit decreases with k at $k = 1 - r$.

The equilibrium capacity is therefore given by $\hat{k}^*(r) = 0$ if $r \leq [0, \underline{r}(c)]$, $\hat{k}^*(r) = k_1(r)$ if $r \in (\underline{r}(c), \overline{r}(c))$, and $\hat{k}^*(r) = k_2$ if $r \in [\overline{r}(c), 1]$. The maximum capacity is installed for $\hat{r}^*$ solving

$$\frac{dk_1(r)}{dr} = \frac{1}{2}\left(\frac{c}{r^2} - \frac{1}{2}\right) = 0;$$

i.e., $\hat{r}^* = \sqrt{2c}$. (Note that $d^2k_1(r)/dr^2 = -c/r^3 < 0$.) The maximum capacity is

$$k_1(\hat{r}^*) = \frac{1}{2}\left(1 - \sqrt{2c}\right) > \hat{k}_2.$$

Hence a binding price increases expected surplus.

As shown in Section 6 the optimal capacity is $k^W = 1 - \sqrt{2c} = 2\hat{k}^*(\hat{r}^*)$. Hence a price cap alone is unable to provide incentives for the monopolist to install the optimal level of capacity. In fact, price caps provide worse incentives for capacity investment and generate a lower expected surplus under full capacity utilization that when the monopolist can withhold capacity as Figure 10 shows. (Note that with capacity withholding the maximum expected surplus that can be realized with an optimal price cap is no less than $\bar{S}(r^*(c))$.)

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